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MATHEMATICAL MODELING OF DISPLACEMENT OF OIL BY WATER UNDER A CYCLIC ACTION ON THE CRACKED-POROUS STRATUM

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A mathematical model is proposed for studying unsteady processes of displacement of oil by water and evaluating the efficiency of a cyclic action on the stratum. A numerical model is presented. An analysis of calculation results is given. An optimum value of the period of the action is obtained that is in agreement with oil-field experimental data.

Introduction. Oil reserves in strata with a cracked-porous structure are among those difficult to recover. In such collectors, oil is concentrated in blocks, and liquid filtration in exploitation proceeds along cracks. If the blocks are hydrophobic, conventional quasisteady waterflooding of the stratum in principle cannot be efficient. In this case, the mechanism of capillary impregnation [1, 2] is inoperative, and practically water alone is filtered along cracks.

Special oil-field experiments showed (see, for example, [3-5]) that a cyclic action on cracked-porous strata significantly decreases the watering of producing wells. This effect has not yet found a proper explanation within the framework of the theory of two-phase filtration. Thus, it was established in [6] that, within the framework of the usual assumptions of the theory of two-phase filtration, "periodic elastic vibrations of liquids in a heterogeneous stratum do not lead ... to a change in the technological characteristics of the exploitation" (p. 58). Tsinkova [6] proposes describing the mechanism of "equalization of saturation under a cyclic action" using hysteresis of phase permeabilities. However, with this approach, "the quantitative scope of the manifestation of the considered effect of equalization of saturation is very narrow" (p. 66).

Below, a mathematical model is proposed that permits studying of unsteady processes of displacement of oil by water and making an evaluation of the efficiency of a cyclic action on the stratum that is in agreement with available oil-field experimental data. A numerical model is presented that approximates the formulated problem. An analysis of computational experiments is given.

Mass Transfer between Blocks and Cracks. A key point in the construction of the model is the description of the flows between blocks and cracks. The calculation of the flows in the general case requires the solution of a complex internal problem of two-phase filtration in blocks in an elastic mode. The boundary conditions in it were specified nonstationary values of the saturation and pressure in cracks. The problem ought to be solved with allowance for capillary barriers (end effects [1]) at the block–crack border. This approach would lead to a double-level model, at whose upper (outer) level there are equations of two-phase filtration in cracks with mass sources that are integral characteristics of the solution of the internal problem for a block.

Instead, in this work we use a rough description of the flows that is based on the following reasoning. Generally speaking, the flows arise for two reasons: first, because the capillary equilibrium is upset, and second, because the elastic reserve of the liquid in the medium changes. In the first case, the surface energy is minimized with the porosity of the medium and the fluid densities unchanged. Here, persistent capillary redistribution of the water and oil proceeds both inside the blocks and between the blocks and the cracks, and selective impregnation of the blocks with the wetting phase occurs. Hydrophobic blocks absorb rather than give off oil, which only hampers the manifestation of the discussed effect. In the second case, the flows compensate

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for the change in the elastic reserve. Here, the overall volume of water and oil that passes from a block to cracks per unit time in unit pore volume of the medium (the flow intensity) is

$$q_\Sigma = -\bar{\beta}\frac{\partial P}{\partial t}. \tag{1}$$

The use of the one-pressure approximation in expression (1) is quite justified. Indeed, evaluations indicate [7] that the time of pressure equalization in the cracks and the blocks is negligible in comparison with the periods of pressure variations that are employed under oil-field conditions. Therefore, in describing the process, the pressure in the cracks and the blocks can be assumed to be the same. The intensities of the water and oil flows are defined separately by the following expressions:

$$q_i = \lambda_i q_\Sigma, \quad i = 1, 2. \tag{2}$$

In liquid filtration from the cracks to the blocks, $\lambda_1(S)$ is determined by the water and oil mobilities in the cracks, and in liquid filtration from the blocks to the cracks, by their mobilities in the blocks.

Mass Transfer in Cracks. The filtration in the cracks is described by the balance equations for the water and oil masses with account for expressions (2) for the flows and by the equations of motion of the fluids [2]:

$$\frac{\partial (m\rho_i S_i)}{\partial t} + \mathrm{div}\,(\rho_i \bar{U}_i) = \rho_i q_i, \tag{3}$$

$$U_i = -\frac{k(P)}{\mu_i} k_i^*(S_i)\,\nabla P, \quad i = 1, 2. \tag{4}$$

The absolute permeability k and the porosity m of the cracks are substantially dependent on the pressure [7]. The functions $k(P)$ and $m(P)$ must be determined experimentally. The dependence $k(P)$ can be determined from the curve of pressure restoration on the producing well after its shutdown [7], and $m(P)$ is determined by the block strains and the structure of the crack assemblage. For example, for a system of identical vertical cracks with the opening h and the surface Σ of the blocks in unit volume of the medium, we have $m = \Sigma h$. It is shown in [7] that upon passage of this crack system from a certain initial state of it with the parameters h_0 and P_0 to the state with h and P, the increment in the crack opening is proportional to the change in pressure: $(h - h_0) = (P - P_0)\nu/E$. Hence,

$$\frac{m - m_0}{m_0} = \frac{l\nu}{h_0 E}(P - P_0). \tag{5}$$

According to data of [7, 8], $l/h \approx 10^3$, $\nu \approx 0.25$, $E \approx 10^5$ atm, $\beta_2 \approx (1\text{–}3)\cdot 10^{-4}$ 1/atm, and $\bar{m} = 0.3$. Then, in accordance with relation (5),

$$\frac{\partial m}{\partial t} = m_0 \varepsilon \frac{\partial P}{\partial t}. \tag{6}$$

Here, the elastic capacity of the cracks is $\varepsilon = 2.5\cdot 10^{-3}$ 1/atm, and $m_0\varepsilon$ turns out to be of the same order of magnitude as β in Eq. (1). This means that the change in the elastic capacity of the cracks must be taken into account in Eq. (3). Clearly, an actual crack assemblage in a stratum is a mixture of ideal crack systems with various values of l/h and orientations. For it, dependence (6) can be adopted only in a first approximation, and therefore, the value of ε becomes only tentative.

Taking into account the aforesaid and using the elastic capacity β_i^* of the cracks with the i-th liquid, Eqs. (3) and (4) are transformed to the form

$$m\beta_i^* S_i \frac{\partial P}{\partial t} + m \frac{\partial S_i}{\partial t} = \mathrm{div}\left(\frac{k_i^*}{\mu_i} k \nabla P\right) + q_i\,, \quad i = 1, 2\,. \tag{7}$$

It should be noted that the phase permeabilities for the blocks and the cracks are markedly different. This is linked with the fact that, in the cracks, the role of capillary forces is diminished and (due to a significant difference in the oil and water viscosities) conditions are set up for displacement of oil by water with a developed tongue formation, which leads to a linear dependence of the phase permeabilities on the saturation [2]. Thus, for the cracks it is possible to use phase permeabilities in the form

$$k_1^* = \begin{cases} 0\,, & 0 \le S \le S_*\,, \\ (S - S_*)/(1 - S_*)\,, & S_* \le S \le 1\,; \end{cases}$$

$$k_2^* = \begin{cases} 1 - S/S^*\,, & 0 \le S \le S^*\,, \\ 0\,, & S^* \le S \le 1\,. \end{cases} \tag{8}$$

Balance of Liquid Masses in Blocks. It is assumed that there is no macroscopic filtration flux in the blocks, and mass transfer in the cracks occurs by elastic forces. In this case, the change in the water and oil content in the blocks is determined by the equations [2]

$$\frac{\partial\,(\rho_i \bar{m}\, \bar{S}_i)}{\partial t} = -\,\rho_i\, q_i\,,$$

which are transformed to the form

$$\bar{m} \frac{\partial \bar{S}_i}{\partial t} + \bar{\beta}_i \bar{S}_i \frac{\partial P}{\partial t} = -\,q_i\,, \quad i = 1, 2\,. \tag{9}$$

The relative phase permeabilities of the blocks are defined by the relations

$$\bar{k}_1^* = \begin{cases} 0\,, & 0 \le \bar{S} \le \bar{S}_*\,, \\ ((\bar{S} - \bar{S}_*)/\bar{S}_*)^3\,, & \bar{S}_* \le \bar{S} \le 1\,; \end{cases}$$

$$\bar{k}_2^* = \begin{cases} ((\bar{S}^* - \bar{S})/(\bar{S}^* - \bar{S}_*))^3\,, & 0 \le \bar{S} \le \bar{S}^*\,, \\ 0\,, & \bar{S}^* \le \bar{S} \le 1\,. \end{cases} \tag{10}$$

The system of equations (1), (2), and (7)-(10) along with the conditions $S_1 + S_2 = 1$ and $\bar{S}_1 + \bar{S}_2 = 1$ describes a substantially unsteady-state mode of filtration in a cracked-porous stratum under conditions of local pressure equilibrium between the blocks and the cracks.

Mathematical Formulation. Let us introduce the filtration rate for the total flux $\vec{U} = \vec{U}_1 + \vec{U}_2$. Upon simple transformation, the system of equations (1), (2), and (7)-(10) is represented as

$$\beta \frac{\partial P}{\partial t} + \mathrm{div}\,\vec{V} = q_\Sigma\,, \quad \vec{V} = -\frac{k}{\mu_1} k^* \nabla P\,,$$

$$m \frac{\partial S}{\partial t} + \beta_1^* S \frac{\partial P}{\partial t} + \mathrm{div}\,(f \cdot \vec{V}) = \lambda q_\Sigma\,, \quad \bar{m} \frac{\partial \bar{S}}{\partial t} + \bar{\beta}_1^* \bar{S} \frac{\partial P}{\partial t} = -\,\lambda q_\Sigma\,, \tag{11}$$

where

$$\lambda = \begin{cases} f(S), & \text{if} \quad \partial P/\partial t > 0, \\ \bar{f}(\bar{S}), & \quad \partial P/\partial t < 0; \end{cases}$$

$$\beta = \beta_1^* S + \beta_2^* (1 - S); \;\; \beta_i^* = m(\alpha + \beta_i);$$

$$k^* = k_1^* + \mu k_2^*; \;\; \mu = \mu_1/\mu_2; \;\; f(S) = k_1^*/k^*; \;\; \bar{f}(\bar{S}) = \bar{k}_1^*/\bar{k}^*.$$

To solve the system of equations (11), boundary and initial conditions must be specified. The initial state of the stratum is determined by the conditions

$$P|_{t=0} = P^0, \;\; S|_{t=0} = S^0 \geq S_*, \;\; \bar{S}|_{t=0} = \bar{S}^0 \geq \bar{S}_*. \tag{12}$$

In the case in question, the region of solution of the problem is multiply connected because of the presence of wells that dissect the stratum. To model a periodic action on the stratum, boundary conditions on a well of radius r_w are specified in the form

$$q(t) = A(1 - \cos(\omega t)) \;\; \text{or} \;\; P_c(t) = P^0 - A(1 - \cos(\omega t)) \tag{13}$$

depending on whether the well operates in the mode of an assigned discharge $q(t)$ or pressure $P_w(t)$ on the well. Here, the period $A > 0$ is for producing wells and $A < 0$ is for pressure wells.

In the practice of stratum exploitation, producing wells are used only up to a certain watering $\hat{H}^*$ of them that represents the limiting portion of water in the well discharge, expressed in percent. Thereupon, the well is closed, i.e., its discharge becomes equal to zero irrespective of its operating mode up to this moment.

At the outer boundary of the stratum, boundary conditions of either the first or second kind for the pressure can be assigned.

The above mathematical model (11) differs substantially from the existing model of a double-porosity medium [1]. The system of corresponding equations for filtration in cracks and blocks can be written in the form [9]

$$\beta \frac{\partial P}{\partial t} + \operatorname{div} \vec{V} = q_\Sigma, \;\; \bar{\beta} \frac{\partial \bar{P}}{\partial t} + \operatorname{div} \vec{\bar{V}} = -q_\Sigma, \;\; q_\Sigma = -\alpha (P - \bar{P}),$$

$$m \frac{\partial S}{\partial t} + \beta_1^* S \frac{\partial P}{\partial t} + \operatorname{div}(f\vec{V}) = \lambda q_\Sigma, \;\; \bar{m} \frac{\partial \bar{S}}{\partial t} + \bar{\beta}_1^* \bar{S} \frac{\partial \bar{P}}{\partial t} + \operatorname{div}(\bar{f}\vec{\bar{V}}) = -\lambda q_\Sigma. \tag{14}$$

The new model is free of a major drawback of the model of a double-porosity medium (14) that is associated with the need to specify the coefficient α of transfer between the cracks and the blocks due to the difference in their pressures (experimental oil-field methods of determining this parameter are lacking so far). At the same time, allowance for the pressure difference and the related transfer coefficient cannot, from our viewpoint, lead to a qualitative distinctiveness of the character displacement of oil by water in a cracked-porous stratum, which is supported by our investigations. Furthermore, the new model is much simpler in its numerical implementation.

Numerical Algorithm. Since the pressure in the vicinity of a well has a logarithmic character, in this vicinity of radius $\tilde{r}$ we convert to the logarithmic coordinate system $\rho = \ln(r/r_w)$. In the region $\rho \in [0, \bar{\rho} = \ln(\tilde{r}/r_w)]$, we introduce a uniform grid D_{h_1} with the step $h_1 = \bar{\rho}/n_1$. In this grid, Eq. (11) for the total flux in the cracks, which is transformed to the logarithmic coordinate system, is approximated by a conservative difference equation. In the region $r \in (\tilde{r}, R)$, a uniform grid D_{h_2} with the step $h_2 = (R - \tilde{r})/n_2$ is constructed. Here, the grid node $\tilde{r}$ is referred to the grid region D_{h_2}. Then, for the difference scheme to be conservative over the entire region, the flux through the left boundary r_w exp $(\bar{\rho} - 0.5h_1)$ of the unit cell that contains the node $\tilde{r}$ must be equal in magnitude to the flux through the boundary $\bar{\rho} - 0.5h_1$ of the grid D_{h_2}. The resulting system of linear equations for the pressure has a three-diagonal matrix and is solved using the factorization method.

Difference equations for the water saturation S must be constructed with allowance for the following factors:

1. The crack porosity is much lower than the block porosity, and the permeability is higher, which leads to a high velocity of water motion in the cracks. The situation is aggravated by the linearity of the phase permeabilities, i.e., by the absence of a saturation jump and rapid spreading of small values of S. All this indicates that the saturation S in the cracks must be calculated using an implicit scheme.

2. The water motion in the blocks is appreciably slower. Here, the time step of the grid is small because of the periodic action on the stratum. This implies that it is expedient to use an explicit scheme in the blocks.

3. In the approximation of the elastic forces in the corresponding equations of system (11), the time layer on which the saturation should be taken is determined by the "sign" of the process: with compression $(\partial P/\partial t > 0)$, the saturation S is taken on the considered layer, otherwise it is taken on the preceding layer.

4. In the approximation of the flows between the blocks and the cracks, the saturation is taken from the considered layer if the flow proceeds from the cracks to the blocks, and from the preceding layer for flow from the blocks to the cracks.

It should be noted that calculation of the saturation from Eqs. (11) in the logarithmic coordinate system and use of the same grid D_{h_1} would lead to a very small time step because of the small volume of unit cells adjacent to the crack. Therefore, in the region $\rho \in (0, \bar{\rho})$, only one grid node is taken, and the unit cell that contains it has the boundaries $0.5h_1$ and $\bar{\rho} - 0.5h_1$. This permits determination of the total flux at these boundaries using nodes of the grid D_{h_1}.

Calculation of the total flux $\vec{V}$ through the boundaries of the unit cells requires the values of the saturation S at the seminodes. For $r \in (\tilde{r}, R)$, it is calculated from equations that are obtained using homographic interpolation [10] of the mean-integral values at the grid nodes, and for $\rho \in [0, \bar{\rho}]$, with the aid of linear interpolation in the radial coordinate system r. Use of this coordinate system rather than the logarithmic one stems from the behavior of the function S, which, unlike the pressure, is not logarithmic.

In the calculation of the water portion in the flux at the seminodes, a scheme of the "corner" type is used in the balance equation for the saturation S in the cracks.

After the pressure and saturation on each time layer have been determined, all needed characteristics of exploitation are calculated both for the stratum as a whole and individually for the cracks and the blocks (the flows between the blocks and the cracks, the well watering, the running and final oil yield, etc.).

As a check of the calculations, the oil imbalance is determined (on each time layer and at the considered instant), which is the difference between the quantities of oil remaining in the stratum that are calculated, on the one hand, in terms of the mean-integral values in the unit cells and, on the other, in terms of the discharge of the producing well.

To implement the two models (11) and (14), corresponding algorithms and programs were worked out that yielded the numerical results presented below.

Analysis of Computational Experiments. Since the work aims at studying the effect of a cyclic action on a stratum, the case of radial filtration in the vicinity of a producing or pressure well that operates in a periodic mode is of greatest interest. Therefore, consideration is subsequently given to such filtration. In this case, the external boundary of the region of solution of the problem is the feed contour $r = R$, on which the pressure P_R is specified. Computational experiments were carried out in three directions:

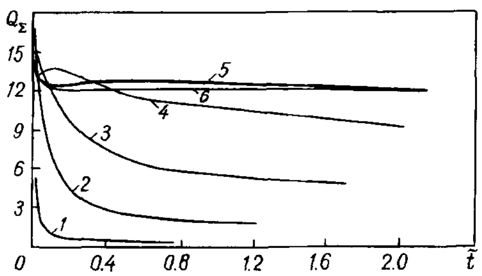
1) study of the effect of the magnitude of the period on the filtration;

2) parametric analysis of the dependences of the oil yield of the stratum and the volume of the liquid extraction (injection) on the elastic capacities of the blocks and the cracks;

3) comparison of results calculated using the above two models.

TABLE 1. Characteristics of the Stratum Exploitation as Functions of the Period T

No. of line	T, h	t^*, days	$\tilde{t}^*$	η, %	$\bar{\eta}$, %	η_Σ, %
1	–	921	0.53	75.3	0.1	8.3
2	720	1290	0.75	80.0	2.6	11.1
3	192	2072	1.2	78.6	9.7	17.2
4	96	2932	1.70	74.6	16.2	22.6
5	48	3454	2.00	72.5	20.2	25.9
6	24	3663	2.12	72.1	20.8	26.4
7	18	3650	2.11	72.4	20.1	25.8



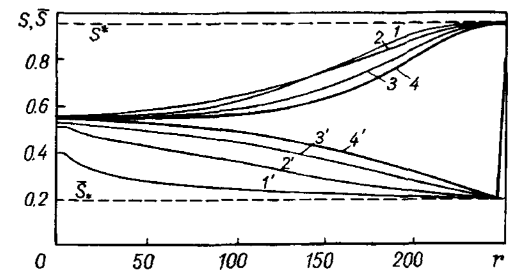


Fig. 1. Overall flows Q_Σ (m^3) between cracks and blocks as functions of dimensionless time $\tilde{t}$ for T = 720, 192, 96, 48, 24, and 18 h (1-6, respectively).

Fig. 2. Distribution of saturation along the stratum in cracks (1-4) and blocks (1′-4′) at the end of the exploitation for T = 720, 192, 96, and 18 h, respectively. r, m.

The following parameters were used in subsequent computational experiments: r_w = 0.1 m, R = 250 m, k = 0.5, μ_1 = 1 mPa·sec, μ_2 = 20 mPa·sec, S_* = 0.02, S^* = 0.95, $\bar{S}_*$ = 0.2, $\bar{S}^*$ = 0.8, m = 0.02, $\bar{m}$ = 0.2, P^0 = 18 MPa, P_R = 18.01 MPa, A = 25 m^3/day, S^0 = 0.02, $\bar{S}^0$ = 0.2, and $\hat{H}$ = 96%. The parameters of the numerical model (the radius of the internal region, the number of nodes in the internal and external regions, the number of time layers in a period, etc.) were varied over a wide range to evaluate the computational error.

Study of the effect of the period. Table 1 presents some calculation results for different values of the period of vibrations T for the end of the stratum exploitation, which corresponds to the instant at which the running value of the watering of the well reaches the limiting value $\hat{H}^*$. The first line corresponds to a steady-state mode of the well.

As the analysis of the experiments revealed, the water-free period and the water-free oil yield under both the steady and cyclic actions remain very small, which is linked with rapid water breakthrough into the well along cracks. Subsequently, however, the process unsteadiness gives rise to mass transfer between the cracks and the blocks along almost entire length of the stratum, which slows down the increase in the well watering $\hat{H}$. The watering decrease is persistent (see Table 1) and leads to a significant increase in the final oil yield η_Σ of the cracked-porous stratum. With decrease in the period of the action, the final oil yield rises, but here the volume of the extracted liquid increases, as indicated by the data of the fourth column. For example, for T = 24 h, the value of $\tilde{t}^*$ increased fourfold and the oil yield η_Σ increased nearly threefold as compared with the steady-state case. As was to be expected, the final oil yield increases due to the blocks, whereas in the cracks it even decreases somewhat. The latter is explained by the fact that the flows between the cracks and the blocks occur along the entire length of the stratum, and the oil that comes from the blocks reduces the oil yield of the cracks. The oil yield η_Σ is maximum at T equal to 24 h. It should be noted that the range of T in which the oil yield is close to maximum is fairly wide.

The period T of the vibrational mode has a substantial effect not only on the basic characteristics of the stratum exploitation but also on the character of the filtration. Figure 1 shows time dependences for the overall flows Q_Σ, which are defined as integral characteristics along the stratum length, for different values of

TABLE 2. Effect of the Model Parameters on the Characteristics of the Stratum Exploitation

No. of line	$\beta_1 \cdot 10^3$	$\beta_2 \cdot 10^3$	$\varepsilon \cdot 10^3$	m	$\tilde{t}^*$	η, %	$\bar{\eta}$, %	η_Σ, %
1	0.03	0.3	3	0.02	2.12	72.1	20.8	26.4
2	0.3	0.3	3	0.02	2.14	72.3	21.0	26.6
3	0	0.3	3	0.02	2.11	72.1	20.7	26.3
4	0.3	0	3	0.02	1.78	76.1	15.1	21.7
5	0.03	0.3	0	0.02	2.22	69.3	27.6	32.1
6	0.03	0.3	3	0.005	2.12	69.5	25.2	26.6

TABLE 3. Comparison of Results Calculated Using the Two Models

No. of line	T, h	$\hat{H}$, %	η, %	$\bar{\eta}$, %	η_Σ, %
1	192	95.3	77.2	8.7	16.2
1′	192	95.2	78.3	9.0	16.5
2	96	94.1	72.0	11.9	18.5
2′	96	94.3	74.5	11.4	18.3
3	48	93.3	68.7	13.6	19.6
3′	48	93.8	72.7	12.5	19.1
4	24	93.2	68.6	13.2	19.2
4′	24	93.6	72.2	12.7	19.2
5	18	93.3	69.6	12.5	18.8
5′	18	93.7	72.2	12.7	19.1

the period T. There is no difficulty in seeing that the flow intensity for T = 18 h is lower than for T = 24 h. It is exactly this that led to a decrease in the oil yield of the stratum.

Figure 2 illustrates the distribution of the saturation S along the stratum in the cracks and the blocks. Here, curves 4 and 4′ correspond to the variants of the calculations with T = 18, 24, and 48 h, which very close saturation distributions. It should be noted that, in the vicinity of the well, the saturations in the blocks and the cracks turned out to be almost the same only for these variants. With increase in T, the water saturation in the blocks decreases sharply. Interestingly, in the vicinity of the feed contour, through which water arrives at the stratum, the displacement of oil in the blocks remains small for any value of the vibration period T. This is explained by the fact that on the feed contour, a constant pressure is maintained and elastic forces practically do not operate.

Parametric analysis. Table 2 presents some results for the oil yield of the stratum and the injection volume as functions of the elastic capacities of the water β_1, the oil β_2, and the cracks ε for the optimum period T = 24 h. An analysis reveals that the elastic capacity of the water β_1 has practically no effect on the filtration (see lines 1, 2, and 3). As regards the elastic capacity of the oil β_2, its magnitude affects the final oil yield η_Σ and the volume of the injected water: the larger β_2, the larger η_Σ and $\tilde{t}^*$ (lines 2 and 4). Here, the oil yield of the blocks $\bar{\eta}$ increases substantially with a simultaneous decrease in the oil yield of the cracks η. In contrast to the elastic capacity of the oil β_2, the effect of the elastic capacity of the cracks ε on the oil yield η_Σ is opposite, that is, the larger ε, the smaller η_Σ and $\bar{\eta}$ with an increase in η, and $\tilde{t}^*$ changes slightly (see lines 1 and 5).

Taking into account that under actual conditions it is very problematic to divide the stratum porosity into the block porosity and the appreciably smaller crack porosity, special computational experiments were conducted. They showed that porosity variations not change the results qualitatively, but only quantitatively. For example, a decrease in the crack porosity by a factor of 4 did not change the optimum period or $\tilde{t}^*$ and slightly increased η_Σ. However, for a smaller m there are a larger oil yield $\bar{\eta}$ of the blocks and a smaller oil yield of the cracks η (lines 1 and 6 in Table 2).

Comparison of results calculated using the two models. Computational experiments based on the above models demonstrated that the qualitative character of the behavior of the saturation and the basic characteristics of the exploitation is the same. Quantitatively, however, the calculation results differ somewhat. Table 3 presents a comparison of some basic characteristics of the stratum exploitation for the same values of the period T with one pore volume of the liquid extracted from the stratum. The lines numbered 1-5 contain results calculated using the new model, and lines $1'-5'$ contain results calculated using model (14).

It should be noted that the flow coefficient α in the model with double porosity affects slightly the qualitative character of the solution, but has a noticeable effect on the quantitative results. Thus, an increase of two orders of magnitude in α in comparison with its value that was specified in the calculation variants of Table 3 increases the total oil yield and the oil yield of the cracks (for example, for T = 18 h, η_Σ = 23.9, η = 59.4, and $\bar{\eta}$ = 19.6). Thus, indeterminacy of the assignment of the parameter α significantly reflected in the final results in evaluating the efficiency of a cyclic action on the stratum.

Conclusions The computational experiments indicate that a cyclic action on a stratum increases its oil yield due to the work of elastic forces, which enhance the mass transfer between the cracks and the blocks, and due to the difference in the phase permeabilities in the blocks and the cracks, which results in the fact that, with a pressure buildup, mainly water arrives at the blocks and, with a pressure reduction, mainly oil is displaced to the cracks. Thus, the proposed mathematical model permits a study of unsteady processes of displacement of oil by water, an evaluation of the efficiency of a cyclic action on the stratum, and a determination of an optimum value of the period T that is in agreement with available oil-field experimental data.

NOTATION

t, time; $\tilde{t}$, dimensionless time expressed in fractions of the pore volume of the stratum ($\tilde{t}$ = 1 corresponds to the time of injection of one pore volume of the liquid); q_Σ, intensity of the flows; S, water saturation; P, total pressure of the fluids in the medium; k, absolute permeability of the crack system; $\bar{\beta} = \bar{\beta}_1^* \bar{S}_1 + \bar{\beta}_2^* \bar{S}_2$, coefficient of elastic capacity of the blocks; $\bar{\beta}_i^* = \beta_m^2 + \bar{m}\beta_i$; β_m and β_i, elastic capacity of the medium and the i-th liquid; S_i, saturation; the overscrore denotes block parameters; the subscripts 1 and 2 refer to water and oil, respectively; $\bar{m}$, block porosity; $\beta_i^* = \varepsilon + m\beta_i$, elastic capacity of the cracks with the i-th liquid; ε, elastic capacity of the cracks; $\lambda_1(S, \bar{S})$, fraction of water in the liquid flux, dependent on the flow direction; λ_2 = $(1 - \lambda_1)$, fraction of oil; k_i^*, ρ_i, μ_i, and U_i, relative phase permeability, density, viscosity, and filtration rate of the i-th phase; h, crack opening; Σ, surface of the blocks in unit volume of the medium; the subscript 0 denotes the parameters of the crack system in a certain initial state of it; l, ν, and E, characteristic dimension, Poisson coefficient, and elastic modulus of the blocks; S_*, $\bar{S}_*$ and S^*, $\bar{S}^*$, coupled and limiting water saturations in the cracks and the blocks, respectively; $\vec{V}$, filtration rate of the total flux; α, coefficient of flow between the cracks and the blocks; r_w, well radius; q, well discharge; r, spatial coordinate; P_w and P_R, pressure on the well at r = r_0 and on the feed contour at $r = R$; $\omega = 2\pi/T$, T, and A, frequency, period, and amplitude of the cyclic action; the superscript 0 denotes the initial distribution of the sought functions at t = 0; ρ = ln (r/r_w), dimensionless spatial coordinate; h_1 and h_2, grid steps in the discrete regions D_{h_1} and D_{h_2}; $\hat{H}$ and $\hat{H}^*$, running and limiting watering; $\tilde{t}^*$ and t^*, dimensionless and dimensional time of stratum exploitation; Q_Σ, overall flows between the cracks and the blocks; η, $\bar{\eta}$, and η_Σ, coefficients of oil yield of the cracks, the blocks, and the entire stratum, respectively.

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